A Note on Basket Securities in Segmented Markets*

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ABSTRACT

Basket securities are securities that bundle several assets and whose payoffs depend on those of the underlying pool of assets, such as index funds and exchange-traded funds (ETFs). I study the design and welfare implications of basket securities issued in markets with limited investor participation in which profit-maximizing intermediaries are involved in financial innovation. I show that when only one intermediary exists, the equilibrium is not constrained efficient. Increasing competition among intermediaries increases the variety of baskets issued but does not necessarily improve investors' welfare.

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Recent decades have seen a substantial increase in financial innovation and in investors' demand for securities that bundle several assets. These "basket securities" have payoffs that depend on those of the underlying pool of assets, such as index funds and ETFs. Despite the importance of these securities, the literature provides limited theoretical guidance on how they develop and what are the welfare implications of their introduction. This paper fills this gap by developing a tractable equilibrium model that explores how and which basket securities emerge in markets with limited investor participation. My findings link institutional features of a market—such as segmentation and gains from trade—to the types of basket securities that emerge in equilibrium.

Bundling securities is redundant in perfect capital markets. In reality, however, asymmetric information, transaction costs, and market incompleteness may explain why investors do not replicate basket securities by themselves.[1] In this paper, I focus on market incompleteness in the sense that investors have limited access to capital markets, as this friction better captures salient characteristics of today's most active markets for basket securities. For example, ETFs—one of the most heavily traded types of securities—are created when a profit-seeking intermediary, typically a large broker or an investment bank, selects a set of stocks and places a large number of their shares in a trust. The trust then issues ETF shares, which the intermediary sells in secondary markets. The intermediary's incentive to offer such securities comes from its exclusive right to change the supply of ETF shares on the market in case arbitrage opportunities exist between the ETF and its underlying basket. For instance, if an ETF is trading at a premium compared to its underlying basket, only its intermediary can create ETF shares, whereas other investors cannot participate in the creation–redemption process and must instead rely on short–long strategies.

The baseline model seeks to capture some of the above market characteristics in a simple form. In particular, the model considers an economy in which profit-seeking intermediaries are involved in financial innovation and investors have limited access to capital markets. Within the ETF example, limited investor participation captures the fact that retail investors cannot participate in the ETF creation–redemption process. The model has three main features. First, there are two assets and two market segments, each populated by a continuum of investors with the same preferences. Second, to capture limited investor participation, investors are not allowed to directly trade across market segments. Third, a

[1]For instance, if investors are asymmetrically informed, a basket security may reduce uninformed investors' trading losses, as the adverse selection costs associated with baskets are typically lower than those associated with individual securities (see, for example, Gorton and Pennacchi (1993) and Subrahmanyan (1991)). In terms of transaction costs, basket securities are desirable because high transaction costs make it expensive for retail investors to replicate diversified portfolios on their own (see, for example, Allen and Santomero (1997)).

profit-seeking intermediary exists who can trade with investors in both market segments: In exchange for shares of investors' assets, the intermediary offers shares of one new security, the basket, which consists of a linear combination of both assets. The intermediary chooses this combination to maximize profits, which depend on investors' demand for the basket and the intermediary's equity stake in the basket. Such an equity stake captures the fact that intermediaries often trade the securities they create.

The first question I address is whether the intermediary would simply issue a basket to complete both market segments. Within the model, the answer varies with the expected payoffs of both assets. If the expected payoff of one asset is sufficiently small compared to the other asset, the intermediary may not find it worthwhile to serve investors in both market segments but instead choose to tailor the basket to one investor type. When designing a basket, the intermediary seeks to maximize profits by increasing trading volume in the basket as well as increasing the expected payoff of its equity stake in the basket. Because the intermediary cares only about its own profits, its incentives may not be aligned with those of investors. Thus, the equilibrium is not always constrained efficient.

Based on this lack of efficiency, I then ask whether introducing competition among intermediaries could improve investors' welfare. I find that if intermediaries compete, different basket securities coexist in equilibrium. All of them, however, are redundant, as they satisfy the same investors' risk sharing needs and are, therefore, equivalent in their spanning role. In sum, introducing competition among intermediaries does not necessarily increase investors' risk sharing opportunities or allow them to attain constrained efficient allocations.

This paper contributes to two strands of the literature. First, it develops a tractable equilibrium model that adds to a body of work focused on understanding security design; see, for example, Allen and Gale (1994), Allen and Santomero (1997), Duffie and Jackson (1989), Ross (1989), Duffie and Rahi (1995), and Rahi and Zigrand (2009, 2010). Unlike these papers, my model focuses on the design of basket securities. Second, this paper adds to a body of work that explores the creation of basket securities; see for example, Gorton and Pennacchi (1990, 1993), Subrahmanyan (1991), and DeMarzo (2005). Unlike these papers, my model explores the creation of basket securities in markets with limited investor participation instead of focusing on settings wherein transaction costs and asymmetric information explain the existence of basket securities.

The rest of the paper is organized as follows. Section I describes the baseline model. Section II analyzes the constrained efficient allocation as a benchmark. Section III solves for the equilibrium and characterizes its properties. Section IV analyzes the effect of competition among intermediaries in the design of basket securities. Finally, section V summarizes the main results. The derivations of formulas, unless otherwise stated, appear in the Appendix.

I. Baseline Model

A. The environment

Consider a single-period economy with one consumption good. Two assets, indexed by $i = \{1, 2\}$, are traded at the beginning of the period, pay at the end of the period, and are in unit net supply. There are two market segments, each populated by a continuum of investors with constant absolute risk aversion (CARA) utility and a risk aversion coefficient $0 \leq \gamma < \bar{\gamma}$. Prior to trading, investors in segment one, hereafter *investors one*, are endowed with all asset 1, whereas investors in segment two, hereafter *investors two*, are endowed with all asset 2. Let $\widetilde{x}_i$ denote the (random) payoff of asset i.

ASSUMPTION 1: *The payoffs of assets jointly follow*

$$\begin{bmatrix} \widetilde{x}_1 \\ \widetilde{x}_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right), \tag{1}$$

where $\mu_i > 0$ and $\sigma_i > 0$ denote the mean and standard deviation of the payoff of asset i. Parameter ρ denotes the correlation between the payoffs of both assets.

To capture market segmentation, direct trading across market segments is not allowed. However, one profit-seeking intermediary exists who can trade with investors in both market segments. When trading, the intermediary offers shares of a newly created security to investors in both market segments in exchange for shares of their assets. As I focus on the design of basket securities, I restrict my analysis to the case in which the newly created security is a linear combination of both assets. If $\widetilde{b}$ denotes the (random) payoff of the newly created security, then

$$\widetilde{b} = \sum_{l=1}^{2} \alpha_l^b \widetilde{x}_l, \tag{2}$$

where $\alpha_i^b \in (0, 1)$ represents the fraction of asset i in the basket—which is selected by the intermediary to maximize profits. For simplicity, I assume that issuing several basket securities may prove too costly for the intermediary, as it may need both first-class distribution channels and time to market each new security among investors across different market segments.[2]

Because investors cannot trade directly across market segments, the intermediary earns an exogenous margin $0 \leq \theta < \bar{\theta}$ per each share of the basket traded in equilibrium. Parameter

[2]As a consequence, the intermediary issues at most one new security in equilibrium.

θ represents the effective marginal segmentation investors encounter when trading across different market segments. Parameter θ also relates to the potential intermediation benefits the intermediary obtains when issuing a basket security in segmented markets. As θ tends toward zero, investors trade at almost no cost across market segments and the equilibrium allocation tends to be Pareto optimal, providing perfect risk sharing. However, as θ moves away from zero, trading across market segments gets costly, but potentially more profitable for the intermediary, and the equilibrium allocation may not be Pareto optimal, as investors' marginal valuations are not equalized in equilibrium.

B. Agents

B.1. Investors

Suppose the intermediary offers shares of a basket that contains a fraction α_i^b of asset i, with $i = \{1, 2\}$. Given an intermediary's offer (α_1^b, α_2^b), let $b_i = b_i(\alpha_1^b, \alpha_2^b)$ denote the fraction of the basket *investors* i buy in exchange for a fraction $\widehat{\alpha}_i^b$ of asset i. Let $\widetilde{c}_i$ denote the consumption of *investors* i after trading. The optimal portfolio of *investors* i, $\left\{b_i^*(\alpha_1^b, \alpha_2^b), \widehat{\alpha}_i^{b*}\right\}$, solves

$$\begin{aligned} \max_{b_i, \widehat{\alpha}_i^b} \quad & \mathbb{E}\left[-e^{-\gamma \widetilde{c}_i}\right] \\ st. \quad & \widetilde{c}_i = (1 - \widehat{\alpha}_i^b)\widetilde{x}_i + b_i \left(\left[\sum_{l=1}^{2} \alpha_l^b \widetilde{x}_l \right] - \theta \right) \\ & 0 \leq b_i \leq 1. \end{aligned} \tag{3}$$

The last inequality in (3) implies that investors are not allowed to short-sell the basket. Otherwise, investors can complete their market segments at no cost and the intermediary's activity becomes redundant. Despite the fact that this constraint may fail to capture trading patterns in some markets, its existence aims to highlight the importance of the intermediary in markets with limited investor participation.

B.2. The intermediary

Before trading, a risk-neutral intermediary decides the composition of the basket, defined by fractions α_1^b and α_2^b, and the fraction of the basket to hold after trading, denoted by b_{int}, in order to maximize profits. The (random) profits of the intermediary, denoted by

$\widetilde{\pi} = \widetilde{\pi}\left(\alpha_1^b, \alpha_2^b, b_{\text{int}}\right)$, are given by

$$\widetilde{\pi}\left(\alpha_1^b, \alpha_2^b, b_{\text{int}}\right) = \beta\left(\sum_{l=1}^{2}\alpha_l^b\widetilde{x}_l\right)b_{\text{int}} + \theta\sum_{l=1}^{2}b_l(\alpha_1^b, \alpha_2^b), \tag{4}$$

where $\beta \geq 0$ parameterizes the relative importance of the intermediary's equity stake in the basket on the intermediary's profit (*skin in the game*). Such an equity stake captures the fact that intermediaries often trade the securities they create, and, hence, the payoff of the basket affects their profits. Such an equity stake may also arise in dynamic settings in which increments in the basket performance generate future increments in the intermediary's market share, as investors may trade its securities more often. All else being equal, the higher β, the higher the effect of the payoff of the basket on $\widetilde{\pi}$. The second term in the right-hand side of equation (4) captures intermediation profits. As a consequence, the intermediary cares about the expected payoff of the basket as well as the trading volume the basket generates in equilibrium. Thus, the intermediary solves

$$\begin{aligned} \max_{(\alpha_1^b,\alpha_2^b,b_{\text{int}})} \quad & \mathbb{E}\left[\widetilde{\pi}\right] = \beta\left(\sum_{l=1}^{2}\alpha_l^b\mu_l\right)b_{\text{int}} + \theta\sum_{l=1}^{2}b_l^*(\alpha_1^b, \alpha_2^b) \\ st. \quad & 0 \leq \alpha_i^b \leq 1\ ,\ i = \{1,2\} \\ & 0 \leq b_{\text{int}} \leq 1\ , \mathrm{E}\left[\widetilde{\pi}^*\right] \geq 0. \end{aligned} \tag{5}$$

The term $\widetilde{\pi}^*$ denotes the profit of the intermediary evaluated at the basket that maximizes expected profits. Thus, the last restriction in (5) represents the intermediary's participation constraint.

C. *Equilibrium*

In equilibrium, all markets clear and agents maximize their expected utility subject to their respective trading constraints.

DEFINITION 1: *An* ***equilibrium*** *is an array of fractions,* $\{\alpha_1^b, \alpha_2^b, \widehat{\alpha}_1^b, \widehat{\alpha}_2^b, b_1^*, b_2^*, b_{int}^*\}$*, such that*

(E1) Investor's maximization: Given (α_1^b, α_2^b)*,* $\left\{b_i^*(\alpha_1^b, \alpha_2^b), \widehat{\alpha}_i^b\right\}$ *solves (3), with* $i = \{1,2\}$*.*
(E2) Intermediary's maximization: The tuple $(\alpha_1^b, \alpha_2^b, b_{int}^*)$ *solves (5).*
(E3) Market clearing: $\forall\ i = \{1,2\}$*,* $(1-\widehat{\alpha}_i^b) + \alpha_i^b(b_1^* + b_2^* + b_{int}^*) = 1$*, and* $\alpha_i^b = \widehat{\alpha}_i^b$*.*

II. Constrained Efficient Allocations

This section explores constrained efficient allocations as a benchmark. Consider a benevolent planner who needs to allocate resources among the intermediary and investors in both market segments using the same technology as the intermediary. Given two constants u_0 and $\pi_0 \geq 0$, the planner's problem can be restated as

$$\begin{aligned} \max_{(\alpha_1,\alpha_2,b_1,b_2,b_{\text{int}})} \quad & \mathbb{E}\left[\widetilde{c}_1\right] - \frac{\gamma}{2}\text{Var}\left[\widetilde{c}_1\right] \\ st. \quad & \mathbb{E}\left[\widetilde{c}_2\right] - \frac{\gamma}{2}\text{Var}\left[\widetilde{c}_2\right] = u_0 \\ & \mathbb{E}\left[\widetilde{\pi}\right] = \pi_0, \end{aligned} \tag{6}$$

as the payoffs of both assets are normally distributed and investors have CARA utility. A feasible allocation is a vector $\left(e_1^1, e_1^2, e_1^{\text{int}}, e_2^1, e_2^2, e_2^{\text{int}}\right)$ that satisfies $e_1^1 + e_1^2 + e_1^{\text{int}} \leq 1$ and $e_2^1 + e_2^2 + e_2^{\text{int}} \leq 1$, where e_j^i and e_j^{int} denote investors i's and the intermediary's allocation of asset j, respectively. In this setting, $e_1^1 = 1 - \alpha_1 + \alpha_1 b_1$, $e_1^2 = \alpha_1 b_2$, $e_2^1 = \alpha_2 b_1$, and $e_2^2 = 1 - \alpha_2 + \alpha_2 b_2$. A feasible allocation is said to be constrained efficient if it solves problem (6).

Figure 1 depicts the Edgeworth box for problem (6). Because there are three types of agents—*investors one*, *investors two*, and the intermediary—a feasible allocation is represented with two points inside the Edgeworth box. An example of a feasible allocation is depicted in the left panel of figure 1 using points (A_1, A_2) and (B_1, B_2). The right panel of figure 1 shows that constrained efficient allocations may not be unique. In particular, if $\mu_1|A_1 - C_1| + \mu_2|A_2 - C_2| = \pi_0$, $\mu_1|A_1 - B_1| + \mu_2|A_2 - B_2| = \pi_0$, and $\mu_1|A_1 - D_1| + \mu_2|A_2 - D_2| = \pi_0$ then allocations $(A_1, 1 - C_1, C_1 - A_1, A_2, 1 - C_2, C_2 - A_2)$, $(A_1, 1 - B_1, B_1 - A_1, A_2, 1 - B_2, B_2 - A_2)$, and $(A_1, 1 - D_1, D_1 - A_1, A_2, 1 - D_2, D_2 - A_2)$ solve (6).

The first order conditions of problem (6) imply

$$e_1^1 = \frac{1}{\gamma}\left(\mu_1 - \frac{\theta}{\alpha_1}\right) - \rho\frac{\sigma_2}{\sigma_1}e_2^1. \tag{7}$$

It follows from equation (7) and inspection of figure 1 that

$$\frac{e_2^1}{e_1^1} = \frac{\alpha_2 b_1}{1 - \alpha_1 + \alpha_1 b_1} = \tan\left(\frac{\pi}{2} - \rho\frac{\sigma_1}{\sigma_2}\right), \tag{8}$$

which relates the fractions of asset 2 and 1 held by *investors one* in constrained efficient allocations. The following assumption ensures that $\frac{e_2^1}{e_1^1}$ is positive in constrained efficient allocations.

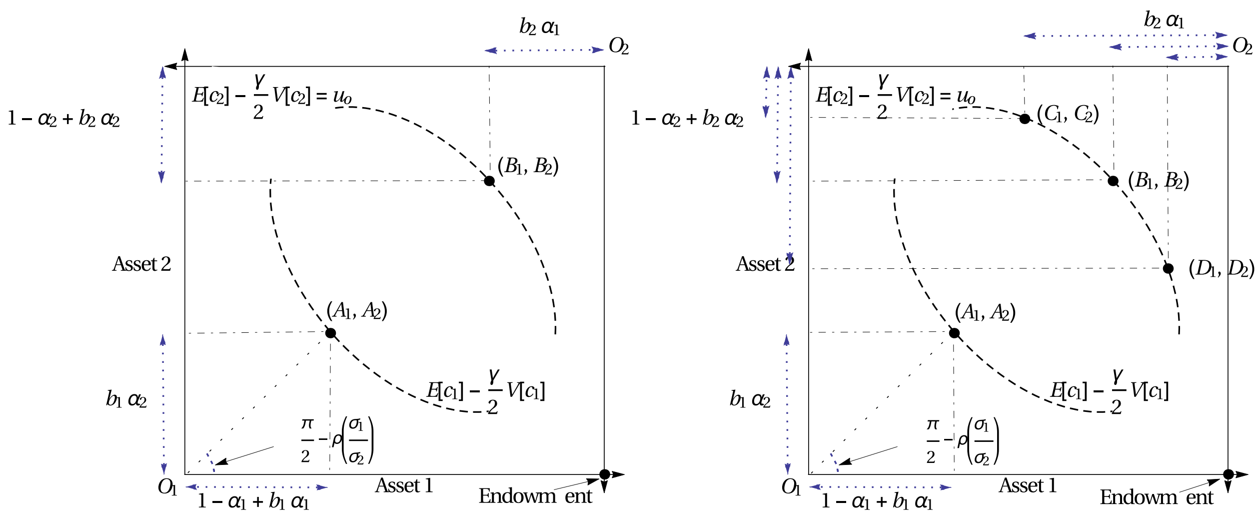

Figure 1. Allocations in the constrained efficient set. In both figures, investors one's quantities are measured with the southwest corner as the origin while investors two's quantities are measured using the northeast corner.

ASSUMPTION 2: $\rho \in \left(-1, \min\left\{1, \frac{\pi}{2}\frac{\sigma_2}{\sigma_1}\right\}\right)$.

It follows from equation (8) that the higher the correlation between the payoffs of both assets, the smaller the ratio $\frac{e_2^1}{e_1^1}$, as investors' risk sharing opportunities and gains from trade decrease if the correlation increases. The same argument applies to the relationship between $\frac{\sigma_1}{\sigma_2}$ and $\frac{e_2^1}{e_1^1}$.

REMARK 1: *Suppose* $\mu_1 = \mu_2$ *and* $\rho\frac{\sigma_1}{\sigma_2} = \frac{\pi}{4}$. *Then,*

$$\frac{e_2^1}{e_1^1} \;=\; \frac{e_2^2}{e_1^2} \;=\; \frac{e_2^{int}}{e_1^{int}} \;=\; 1$$

Thus, constrained efficient allocations provide investors perfect risk sharing. However, both investors indifference curves are not tangent as the expected payoff of the intermediary is π_0.

III. Equilibrium

This section studies the composition of the basket and the allocations that arise from the equilibrium of a market-mediated exchange. I then compare equilibrium with constrained efficient allocations to understand the extent to which the introduction of a basket security in segmented markets provides the right instruments for investors' risk sharing.

A. *Investors i's optimal portfolio*

For a given intermediary's offer (α_1^b, α_2^b), the first order conditions of *investors* i are given by:

$$\left(\sum_{l=1}^{2}\alpha_l^b\mu_l\right) - \theta - \gamma\left([1-\widehat{\alpha}_i^b+\alpha_i^b b_i^*][\alpha_i^b\sigma_i^2+\rho\sigma_i\sigma_j\alpha_j^b]+b_i^*\alpha_j^b[\alpha_i^b\rho\sigma_i\sigma_j+\alpha_j^b\sigma_j^2]\right) \;=\; 0,\; j\neq i$$

Reordering the previous equation yields

$$\underbrace{[1-\widehat{\alpha}_i^b+\alpha_i^b b_i^*]}_{\text{position in asset } i} \;=\; \frac{\left(\sum_{l=1}^{2}\alpha_l^b\mu_l\right)-\theta}{\gamma\sigma_i^2\left[\alpha_i^b+\rho\frac{\sigma_j}{\sigma_i}\alpha_j^b\right]} - \left(\frac{\sigma_j}{\sigma_i}\right)^2\left[\frac{\alpha_j^b+\rho\frac{\sigma_i}{\sigma_j}\alpha_i^b}{\alpha_i^b+\rho\frac{\sigma_j}{\sigma_i}\alpha_j^b}\right]\underbrace{b_i^*\alpha_j^b}_{\text{position in asset } j},\; j\neq i,\; i=\{1,2\} \quad (9)$$

which relates the positions that *investors* i hold in assets i and j after trading. The next assumption ensures that *investors* i's positions in assets i and j are positive.

ASSUMPTION 3: *Parameters $\bar{\gamma}$ and $\bar{\theta}$ are sufficiently small so that for any given intermediary's offer $(\alpha_1^b, \alpha_2^b) \in (0,1)\times(0,1)$, investors i's positions in asset i and j are positive.*

For a given intermediary's offer (α_1^b, α_2^b), it follows from the first order condition of *investors* i that $b_i^*(\alpha_1^b, \alpha_2^b)$ is decreasing in θ, ρ, $\rho\sigma_i\sigma_j$, and γ, and increasing in $\sum_{l=1}^2\alpha_l^b\mu_l$, all else being equal. The higher θ, the higher the cost of trading the basket, and, thus, the lower investors' demand. The higher the correlation between the payoff of both assets, ρ, the lower the gains from trading, and, thus, the lower investors' demand. The same argument applies for the covariance between the payoffs of both assets, $\rho\sigma_i\sigma_j$. The higher investors' risk aversion, γ, the larger the effect of consumption volatility in investors' utility, and, thus, the lower investors' demand, as investing in the basket is intrinsically risky. Finally, the higher the expected payoff of the basket, $\sum_{l=1}^2\alpha_l^b\mu_l$, the higher expected consumption, and, thus, the higher investors' demand.

B. *Equilibrium*

In equilibrium, all markets clear. As a result, $b^*_{\text{int}} = 1 - \sum_{l=1}^{2} b^*_l$ and the equilibrium basket structure (α^b_1, α^b_2) solves

$$\begin{aligned} \max_{(\alpha^b_1,\alpha^b_2)} \quad & \beta\left(\sum_{l=1}^{2}\alpha^b_l\mu_l\right)\left(1-\sum_{l=1}^{2}b^*_l(\alpha^b_1,\alpha^b_2)\right)+\theta\sum_{l=1}^{2}b^*_l(\alpha^b_1,\alpha^b_2) \\ st. \quad & 0\le\alpha^b_i\le 1\ ,\ i=\{1,2\} \\ & \mathrm{E}\left[\widetilde{\pi}^*\right]\ge 0. \end{aligned} \tag{10}$$

The first order conditions of problem (10) are

$$\beta\mu_i\left(1-\sum_{l=1}^{2}b^*_l\right) = \beta\left(\sum_{l=1}^{2}\alpha^b_l\mu_l-\theta\right)\sum_{l=1}^{2}\frac{\partial b^*_l}{\partial\alpha^b_i},\quad i=\{1,2\}\,. \tag{11}$$

Thus, in equilibrium the composition of the basket equalizes the marginal gains generated by the intermediary's equity stake in the basket—represented in the left hand side of (11)—with the marginal gains generated by investors that trade the basket more frequently—represented in the right hand side of (11). It directly follows from the system of equations (11) that

$$\left(\frac{\sum_{l=1}^{2}\frac{\partial b^*_l}{\partial\alpha^b_2}}{\sum_{l=1}^{2}\frac{\partial b^*_l}{\partial\alpha^b_1}}\right) = \frac{\mu_2}{\mu_1}, \tag{12}$$

which can be rewritten as

$$\underbrace{\log\left(\sum_{l=1}^{2}\frac{\partial b^*_l}{\partial\alpha^b_2}\right)-\log\left(\sum_{l=1}^{2}\frac{\partial b^*_l}{\partial\alpha^b_1}\right)}_{\Delta\text{ Trading volume}} = \underbrace{\left(\log\left(\mu_2\right)-\log\left(\mu_1\right)\right)}_{\Delta\text{ Skin in the game}}. \tag{13}$$

In sum, changes in the composition of the basket are driven by either changes in the basket trading volume or changes in the intermediary's equity stake in the basket. It follows from equation (9) that changes in trading volume are driven by changes in (a) investors' risk preferences, γ; (b) market segmentation, θ; and (c) investors' risk sharing opportunities—which can be characterized by the tuple $(\rho, \sigma_1, \sigma_2)$; whereas changes in the intermediary's equity stake are driven by changes in the expected payoffs of both assets, as noted in equation (13).

If $\beta = 0$, then the intermediary only profits from trading volume, which is maximized by selling all basket shares. As a consequence, $b^*_{\text{int}} = 0$ and $\sum_{l=1}^{2} b^*_l = 1$, and all baskets $(\alpha^b_1, \alpha^b_2) \in (0,1) \times (0,1)$ yield the same expected profits for the intermediary. Thus, the basket that replicates the market portfolio—a portfolio held by the average investor—is one equilibrium among infinitely many equilibria. If $\beta > 0$ and $\mu_1 = \mu_2$, changes in the equilibrium basket are only driven by changes in trading volume. Finally, if $\beta > 0$ and $\mu_1 \neq \mu_2$, then the intermediary takes into account its *skin in the game* when tailoring the basket to the market segment that generates the highest expected profits. It then follows,

PROPOSITION 1: *If $\beta > 0$, then the basket security does not necessarily replicate the market portfolio, as the intermediary tailors the basket to the market segment that maximizes expected profits. If $\beta = 0$, then the basket security that replicates the market portfolio is one among infinitely many equilibria.*

C. *Efficiency of equilibrium allocations*

Because investors cannot trade directly across market segments, markets are incomplete and, therefore, equilibrium allocations may not be constrained efficient. I thus explore the conditions under which equilibrium and constrained efficient allocations are equal so that introducing a basket security provides the right instruments for investors' risk sharing.

Without loss of generality, consider $i = 1$, as the problem of both investor types is symmetric. It follows from comparing the allocations in equations (7) and (9) that *investors one*'s equilibrium and constrained efficient allocations are equal if the following system of equations is satisfied:

$$\frac{1}{\gamma}\left(\mu_1 - \frac{\theta}{\alpha_1}\right) = \frac{\left(\sum_{l=1}^{2} \alpha_l^b \mu_l\right) - \theta}{\gamma \sigma_1^2 \left[\alpha_1^b + \rho \frac{\sigma_2}{\sigma_1} \alpha_2^b\right]} \quad \text{and} \quad \rho \frac{\sigma_2}{\sigma_1} = \left(\frac{\sigma_2}{\sigma_1}\right)^2 \left[\frac{\alpha_2^b + \rho \frac{\sigma_1}{\sigma_2} \alpha_1^b}{\alpha_1^b + \rho \frac{\sigma_2}{\sigma_1} \alpha_2^b}\right]. \tag{14}$$

The second equation in (14) is satisfied only if $\rho \in \{-1, 1\}$. As $\rho \in \left(-1, \min\left\{1, \frac{\pi}{2}\frac{\sigma_2}{\sigma_1}\right\}\right)$, the allocations attained as the outcome of a market-mediated equilibrium are not constrained efficient. It then follows,

PROPOSITION 2: *If only one intermediary exists, a basket security does not allow investors to attain constrained efficient allocations.*

IV. Competition among Intermediaries

This section analyzes whether introducing competition among intermediaries increases investors' welfare, as the equilibrium in section III is not constrained efficient. In particular, I study the effect of competition among intermediaries on (a) the composition of baskets and (b) investors' welfare.

Consider an economy with $K \geq 2$ risk-neutral intermediaries in which each intermediary issues at most one basket security. Before trading, intermediaries face the following two-stage non-cooperative game. In the first stage, intermediaries choose whether to enter each market segment and then observe who entered each segment. In the second stage, intermediaries select the composition of their baskets to maximize profits. Immediately after, they observe the composition of each basket and choose whether to trade shares of their assets in exchange for shares of the baskets available in each market segment.

Baskets and intermediaries are indexed by $k = \{1, \cdots, K\}$. Let α_{ik}^b denote the fraction of asset i in basket k that intermediary k selects to maximize profits. Suppose intermediary k offers the basket $(\alpha_{1k}^b, \alpha_{2k}^b)$ to both investor types. Let $b_{ik} = b_{ik}(\alpha_{1k}^b, \alpha_{2k}^b)$ denote the fraction of basket k that *investors* i buy from intermediary k in exchange for a fraction $\widehat{\alpha}_{ik}^b$ of asset i. Let θ_k denote an exogenous margin intermediary k earns per each share of its basket traded in equilibrium, and let $b_{\text{int},k}$ denote the fraction of basket k that intermediary k holds after trading. The (random) profits of intermediary k are given by

$$\widetilde{\pi}_k \quad = \quad \beta_k \left(\sum_{l=1}^{2} \alpha_{lk}^b \widetilde{x}_l \right) b_{\text{int},k} + \theta_k \sum_{l=1}^{2} b_{lk}, \tag{15}$$

where $\beta_k \geq 0$ parameterizes the relative importance of intermediary k's equity stake in basket k on intermediary k's profits.

Given intermediary k's offer $\left(\alpha_{1k}^b, \alpha_{2k}^b\right)$, the optimal portfolio of *investors* i, $\left\{b_{ik}^*, \widehat{\alpha}_{ik}^{b*}\right\}_{k=1}^K$, solves

$$\begin{aligned} \max_{\left\{b_{ik}, \widehat{\alpha}_{ik}^b\right\}_{k=1}^K} \quad & \mathbb{E}\left[-e^{-\gamma \widetilde{c}_i}\right] \\ st. \quad & \widetilde{c}_i = \left(1 - \sum_{l=1}^{K} \widehat{\alpha}_{il}^b\right) \widetilde{x}_i + \sum_{l=1}^{K} b_{il} \left(\left[\sum_{s=1}^{2} \alpha_{sl}^b \widetilde{x}_s \right] - \theta_l \right) \\ & 0 \leq b_{ik} \leq 1, \ k = \{1, \cdots, K\}. \end{aligned} \tag{16}$$

Let $\phi > 0$ and $\eta > 0$. The K first order conditions of problem (16) can be solved only if the composition of basket k satisfies[3]

$$\alpha_{jk}^b = \zeta \alpha_{ik}^b \quad \text{and} \quad \alpha_{ik}^b = \lambda \theta_k, \ j \neq i, \tag{17}$$

with $\zeta = \left(\frac{\sigma_i}{\sigma_j}\right)^2 \left(\frac{\eta - \rho \frac{\sigma_j}{\sigma_i}}{1 - \eta \rho \frac{\sigma_i}{\sigma_j}}\right)$, and $\lambda = \frac{1}{\mu_i + \zeta \mu_j - \phi \gamma \sigma_i^2 \left(1 + \rho \zeta \frac{\sigma_j}{\sigma_i}\right)}$. Thus,

[3] See Appendix for more details.

$$b^*_{ik}\begin{cases} > 0 & \alpha^b_{jk} = \zeta\alpha^b_{ik} \text{ and } \alpha^b_{ik} = \lambda\theta_k \\ 0 & \text{otherwise.}\end{cases}$$

In equilibrium all markets clear, and, thus, $\alpha^b_{ik} = \widehat{\alpha}^b_{ik}$ and $b^*_{\text{int},k} = 1 - \sum_{l=1}^2 b^*_{lk}$, $\forall\ i, k$. Provided that

$$\mathbb{E}\left[\widetilde{\pi}^*_k\left(\lambda\theta_k, \eta\lambda\theta_k\right)\right] = \beta_k\left(\mu_i + \eta\mu_j\right)\lambda\theta_k\left(1 - \sum_{l=1}^2 b^*_{lk}\right) + \theta_k\sum_{l=1}^2 b^*_{lk} > 0,\ j \neq i, \quad (18)$$

the basket $\left(\alpha^b_{ik}, \alpha^b_{jk}\right) = \left(\lambda\theta_k, \zeta\lambda\theta_k\right)$ solves intermediary k's maximization problem, as all other feasible baskets yield zero profits. It follows from the system of equations (17) that changes in the composition of basket k are driven by changes in (a) intermediation fees, θ_k; (b) investors' risk sharing opportunities—characterized by the tuple $(\rho, \sigma_1, \sigma_2)$—and (c) the expected payoff of both assets. It is important to note that this result does not rely on intermediaries' risk neutrality.

In sum, introducing competition among intermediaries causes them to issue baskets that are equivalent in their spanning role, as the ratio $\frac{\alpha^b_{jk}}{\alpha^b_{ik}} = \zeta$ is constant across k. Thus, changes in neither $\{\beta_k\}_{k=1}^K$ nor the number of intermediaries alter the composition of basket securities at equilibrium. Furthermore, different baskets coexist in equilibrium if θ_k varies across intermediaries, as the number of shares of both assets in basket k depends on θ_k. It then follows,

PROPOSITION 3: *If intermediaries compete, they issue different basket securities that do not necessarily increase investors' risk-sharing opportunities, as all basket securities are equivalent in their spanning role.*

To assess the efficiency of the equilibrium allocations in settings in which intermediaries compete, I compare equilibrium and constrained efficient allocations, as I did in section III. It follows from the first order conditions of problem (16) that in equilibrium

$$\underbrace{\left[1 - \sum_{l=1}^K\left(1 - b^*_{il}\right)\alpha^b_i\right]}_{\text{position in asset } i} = \frac{\left(\sum_{l=1}^2 \alpha^b_{lk}\mu_l\right) - \theta_k}{\gamma\sigma_i^2\left[\alpha^b_{ik} + \rho\frac{\sigma_j}{\sigma_i}\alpha^b_{jk}\right]} - \left(\frac{\sigma_j}{\sigma_i}\right)^2\left[\frac{\alpha^b_{ik}\rho\frac{\sigma_i}{\sigma_j} + \alpha^b_{jk}}{\alpha^b_{ik} + \rho\frac{\sigma_j}{\sigma_i}\alpha^b_{jk}}\right]\underbrace{\sum_{l=1}^K b^*_{il}\alpha^b_{jl}}_{\text{position in asset } j},\ j \neq i,\ \forall\ k,$$

which relates the fraction of asset i and j that *investors* i hold after trading. It follows from comparing the allocations in the previous equation and equation (7) that *investors one*'s equilibrium and constrained efficient allocations are equal if the following $2 \times K$ equations are satisfied:

$$\frac{1}{\gamma}\left(\mu_1 - \frac{\theta}{\alpha_1}\right) = \frac{\left(\sum_{l=1}^{2}\alpha_{lk}^{b}\mu_l\right) - \theta_k}{\gamma\sigma_i^2\left[\alpha_{1k}^{b} + \rho\frac{\sigma_2}{\sigma_1}\alpha_{2k}^{b}\right]} \quad \text{and} \quad \rho\frac{\sigma_2}{\sigma_1} = \left(\frac{\sigma_2}{\sigma_1}\right)^2\left[\frac{\alpha_{2k}^{b} + \rho\frac{\sigma_1}{\sigma_2}\alpha_{1k}^{b}}{\alpha_{1k}^{b} + \rho\frac{\sigma_2}{\sigma_1}\alpha_{2k}^{b}}\right], \ \forall\, k \quad (19)$$

The second equation in (19) is satisfied only if $\rho \in \{-1, 1\}$. As $\rho \in \left(-1, \min\left\{1, \frac{\pi}{2}\frac{\sigma_2}{\sigma_1}\right\}\right)$, equilibrium allocations are not constrained efficient even after introducing competition among intermediaries. It then follows,

PROPOSITION 4: *If intermediaries compete, basket securities do not allow investors to attain constrained efficient allocations.*

V. Concluding Remarks

I present a tractable equilibrium model of basket securities issued in markets with limited investor participation in which profit-maximizing intermediaries are involved in financial innovation. I find that if only one intermediary exists, the equilibrium is not constrained efficient, as the intermediary's incentives may not be aligned with those of investors. I then analyze how competition among intermediaries affects the equilibrium basket structure and investors' welfare. I find that competition generates the coexistence of several baskets that do not necessarily improve investors' risk sharing opportunities, as all of these baskets are equivalent in their spanning role.

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Appendix: Derivation of Formulas

This section contains the derivations of the formulas presented in the paper. Let $\widetilde{x}_i$ denote the (random) payoff of asset i, $i = \{1, 2\}$. Let α_i^b denote the fraction of asset i in the basket. If $\widetilde{b}$ denotes the (random) payoff of the basket, then $\widetilde{b} = \sum_{l=1}^{2} \alpha_l^b \widetilde{x}_l$. Let $b_i \in [0, 1]$ denote the fraction of the basket that *investors* i buy, $i = \{1, 2\}$ and b_{int} the fraction the basket the intermediary holds after trading. If $\widetilde{\pi}$ denotes the (random) profits of the intermediary, it is assumed that

$$\widetilde{\pi} \quad = \quad \beta \left(\sum_{l=1}^{2} \alpha_l^b \widetilde{x}_l \right) b_{\text{int}} + \theta \sum_{l=1}^{2} b_l \tag{1}$$

where $\beta \geq 0$ parameterizes the intermediary's "skin in the game" and $\theta > 0$ represents a basket transaction fee.

Planner's Problem

Given two constants π_0 and u_0, the planner's problem can be stated as

$$\begin{aligned} \max_{(\alpha_1,b_1,\alpha_2,b_2)} \quad & \mathbb{E}\left[-e^{-\gamma\widetilde{c}_1}\right] \\ st. \quad & \mathbb{E}\left[-e^{-\gamma\widetilde{c}_2}\right] = e^{u_0} \\ & \mathbb{E}\left[\widetilde{\pi}\right] = \pi_0 \end{aligned} \tag{2}$$

where $\widetilde{c}_i = (1-\alpha_i^b)\widetilde{x}_i + b_i\left(\left[\sum_{l=1}^{2}\alpha_l^b\widetilde{x}_l\right] - \theta\right)$, $i = \{1,2\}$. Provided that payoffs are normally distributed and investors have CARA utility, solving problem (2) is equivalent to solving

$$\begin{aligned} \max_{(\alpha_1,b_1)} \quad & \mathbb{E}\left[\widetilde{c}_1\right] - \frac{\gamma}{2}\text{Var}\left[\widetilde{c}_1\right] \\ st. \quad & \mathbb{E}\left[\widetilde{c}_2\right] - \frac{\gamma}{2}\text{Var}\left[\widetilde{c}_2\right] = u_0 \\ & \mathbb{E}\left[\widetilde{\pi}\right] = \pi_0 \end{aligned} \tag{3}$$

A feasible allocation is a vector $\left(e_1^1, e_1^2, e_1^{\text{int}}, e_2^1, e_2^2, e_2^{\text{int}}\right)$ that satisfies $e_1^1+e_1^2+e_1^{\text{int}} \leq 1$ and $e_2^1+e_2^2+e_2^{\text{int}} \leq 1$, where e_j^i and e_j^{int} denote investors i's and the intermediary's allocation of asset j, respectively. In this setting, $e_1^1 = 1-\alpha_1^b+\alpha_1^b b_1$, $e_1^2 = \alpha_1^b b_2$, $e_2^1 = \alpha_2^b b_1$, and $e_2^2 = 1-\alpha_2^b+\alpha_2^b b_2$. A feasible allocation is said to be constrained efficient if it solves (3). The last restriction in (3) implies

$$\beta\left(\sum_{l=1}^{2}\alpha_l^b\mu_l\right)b_{\text{int}} + \theta\sum_{l=1}^{2}b_l = \pi_0 \quad \rightarrow \quad \sum_{l=1}^{2}b_l = \frac{\pi_0 - \beta(\sum_{l=1}^{2}\mu_l\alpha_l^b)b_{\text{int}}}{\theta} \tag{4}$$

Because $e_i^1 + e_i^2 + e_i^{\text{int}} = 1$, $i = \{1,2\}$ then

$$e_i^{\text{int}} = \alpha_i^b - \alpha_i^b\sum_{l=1}^{2}b_l. \tag{5}$$

It follows from (4) and (5) that

$$e_i^{\text{int}} = \alpha_i^b - \alpha_i^b\frac{\pi_0 - \beta(\sum_{l=1}^{2}\mu_l\alpha_l^b)b_{\text{int}}}{\theta}. \tag{6}$$

Because $\mu_1 e_1^{\text{int}} + \mu_2 e_2^{\text{int}} = \pi_0$ then

$$b_{int} = \frac{\theta\pi_0 - (\theta-\pi_0)((\alpha_2^b)^2 + \alpha_1^b\mu_1)}{\beta((\alpha_2^b)^2 + \alpha_1^b\mu_1)(\sum_{l=1}^{2}\mu_l\alpha_l^b)}. \tag{7}$$

The first order conditions of problem (3) can be stated as:

$$\frac{\partial}{\partial e_1^1}\left(\mathbb{E}\left[\widetilde{c}_1\right] - \frac{\gamma}{2}\text{Var}\left[\widetilde{c}_1\right]\right) = \mu_1 - \theta\frac{\partial b_1}{\partial e_1^1} - \frac{\gamma}{2}\left\{2e_1^1\sigma_1^2 + 2\rho\sigma_1\sigma_2 e_2^1\right\} = 0 \rightarrow e_1^1 = \frac{1}{\gamma}\left(\mu_1 - \theta\frac{\partial b_1}{\partial e_1^1}\right) - \rho\frac{\sigma_2}{\sigma_1}e_2^1 \tag{8}$$

$$\frac{\partial}{\partial e_2^1}\left(\mathbb{E}\left[\widetilde{c}_1\right] - \frac{\gamma}{2}\text{Var}\left[\widetilde{c}_1\right]\right) = \mu_2 - \frac{\partial b_1}{\partial e_2^1} - \frac{\gamma}{2}\left\{2e_2^1\sigma_2^2 + 2\rho\sigma_1\sigma_2 e_1^1\right\} = 0 \rightarrow e_2^1 = \frac{1}{\gamma}\left(\mu_2 - \theta\frac{\partial b_1}{\partial e_2^1}\right) - \rho\frac{\sigma_1}{\sigma_2}e_1^1 \tag{9}$$

Because $b_1 = \frac{e_2^1}{\alpha_2}$, equation (9) implies

$$e_2^1 \quad = \quad \frac{1}{\gamma}\left(\mu_2 - \frac{\theta}{\alpha_2}\right) - \rho\frac{\sigma_1}{\sigma_2}e_1^1. \tag{10}$$

It then follows from inspection of the left panel in figure 1 that

$$\frac{e_2^1}{e_1^1} \quad = \quad \frac{\alpha_2 b_1}{1-\alpha_1+\alpha_1 b_1} = \tan\left(\frac{\pi}{2} - \rho\frac{\sigma_1}{\sigma_2}\right) \tag{11}$$

It follows directly from the right panel of figure 1 that if $\mu_1 = \mu_2$ and $\rho\frac{\sigma_1}{\sigma_2} = \frac{\pi}{4}$ then

$$\frac{e_2^1}{e_1^1} \quad = \quad \frac{e_2^2}{e_1^2} \quad = \quad \frac{e_2^{\text{int}}}{e_1^{\text{int}}} \quad = \quad 1$$

Trading equilibrium

Investors' problem

Given (α_1^b, α_2^b), the optimal portfolio of *investors* i, b_i^*, solves

$$\begin{aligned} \max_{b_i, \widehat{\alpha}_i^b} \quad & \mathbb{E}\left[-e^{-\gamma \widetilde{c}_i}\right] \\ st. \quad & \widetilde{c}_i = (1-\widehat{\alpha}_i^b)\widetilde{x}_i + b_i\left(\left[\sum_{l=1}^{2}\alpha_l^b \widetilde{x}_l\right] - \theta\right) \\ & 0 \leq b_i \leq 1 \end{aligned} \tag{12}$$

Provided that assets' payoff are normally distributed and investors have CARA utility, maximizing investors' expected utility is equivalent to maximizing investors' certain equivalent $\mathbb{E}[\widetilde{c}_i] - \frac{\gamma}{2}\text{Var}[\widetilde{c}_i]$. As a consequence, the first order condition of *investors* i are given by:

$$\left(\sum_{l=1}^{2}\alpha_l^b \mu_l\right) - \theta - \gamma\left([1-\widehat{\alpha}_i^b + \alpha_i^b b_i^*][\alpha_i^b \sigma_i^2 + \rho\sigma_i\sigma_j\alpha_j^b] + b_i^*\alpha_j^b[\alpha_i^b\rho\sigma_i\sigma_j + \alpha_j^b\sigma_j^2]\right) \quad = \quad 0, \; j \neq i \tag{13}$$

Reordering (13) yields

$$[1-\widehat{\alpha}_i^b + \alpha_i^b b_i^*] \quad = \quad \frac{\left(\sum_{l=1}^{2}\alpha_l^b\mu_l\right) - \theta}{\gamma\sigma_i^2\left[\alpha_i^b + \rho\frac{\sigma_j}{\sigma_i}\alpha_j^b\right]} - \left(\frac{\sigma_j}{\sigma_i}\right)^2\left[\frac{\alpha_i^b\rho\frac{\sigma_i}{\sigma_j} + \alpha_j^b}{\alpha_i^b + \rho\frac{\sigma_j}{\sigma_i}\alpha_j^b}\right] b_i^*\alpha_j^b, \; j \neq i \tag{14}$$

which relates the fraction of asset i to the fraction of asset j that *investors* i hold after trading.

Intermediary's problem at equilibrium

In equilibrium all markets clear. As a consequence, $b^*_{\text{int}} = 1 - \sum_{l=1}^{2} b^*_l$ and the equilibrium basket structure (α^b_1, α^b_2) solves

$$\max_{(\alpha^b_1,\alpha^b_2)} \quad \beta\left(\sum_{l=1}^{2}\alpha^b_l\mu_l\right)\left(1-\sum_{l=1}^{2}b^*_l\right)+\theta\sum_{l=1}^{2}b^*_l \tag{15}$$
$$st. \quad 0 \le \alpha^b_i \le 1\ ,\ i=\{1,2\}$$
$$\mathrm{E}\left[\widetilde{\pi}^*\right] \ge 0.$$

The first order conditions of problem (15) are given by

$$\beta\mu_i + \left(\sum_{l=1}^{2}\frac{\partial b^*_l}{\partial \alpha^b_i}\right)\left(\theta-\beta\sum_{l=1}^{2}\alpha^b_l\mu_l\right)-\left(\sum_{l=1}^{2}b^*_l\right)\beta\mu_i \quad = \quad 0, \quad i=\{1,2\}\,. \tag{16}$$

which imply that the following equality is satisfied:

$$\frac{\mu_2}{\mu_1} \quad = \quad \left(\frac{\sum_{l=1}^{2}\frac{\partial b^*_l}{\partial\alpha^b_2}}{\sum_{l=1}^{2}\frac{\partial b^*_l}{\partial\alpha^b_1}}\right), \tag{17}$$

which can be rewritten as,

$$\underbrace{\left(\log\left(\mu_2\right)-\log\left(\mu_1\right)\right)}_{\Delta\text{Skin in the game}} \quad = \quad \underbrace{\log\left(\sum_{j=1}^{2}\frac{\partial b^*_j}{\partial\alpha^b_2}\right)-\log\left(\sum_{j=1}^{2}\frac{\partial b^*_j}{\partial\alpha^b_1}\right)}_{\Delta\text{Trading volume.}} \tag{18}$$

Competition among intermediaries

Baskets and intermediaries are indexed by $k = \{1, \cdots, K\}$. Let α^b_{ik} denote the fraction of asset i in basket k selected by intermediary k to maximize her profits. Let $b_{\text{int},k}$ denote the fraction of basket k that intermediary k holds after trading.

Investors' problem

For a given intermediary k's offer $\left(\alpha^b_{1k}, \alpha^b_{2k}\right)$, let b_{ik} denote the fraction of basket k *investors* i buy. The optimal portfolio of *investors* i, $\{b^*_{ik}\}_{k=1}^{K}$, solves

$$\max_{\left\{b_{ik},\widehat{\alpha}^b_{ik}\right\}_{k=1}^{K}} \quad \mathbb{E}\left[-e^{-\gamma\widetilde{c}_i}\right] \tag{19}$$
$$st. \quad \widetilde{c}_i = \left(1-\sum_{l=1}^{K}\widehat{\alpha}^b_{il}\right)\widetilde{x}_i+\sum_{l=1}^{K}b_{il}\left(\left[\sum_{s=1}^{2}\alpha^b_{sl}\widetilde{x}_s\right]-\theta_l\right)$$
$$0 \le b_{ik} \le 1,\ k=\{1,\cdots,K\}$$

The K first order conditions of *investors* i are given by:

$$\left(\sum_{l=1}^{2}\alpha_{lk}^{b}\mu_{l}\right)-\theta_{k}-\gamma\left(\left[\alpha_{ik}^{b}\sigma_{i}^{2}+\rho\sigma_{i}\sigma_{j}\alpha_{jk}^{b}\right]\left[1-\sum_{l=1}^{K}\left(1-b_{il}^{*}\right)\alpha_{i}^{b}\right]+\left[\alpha_{ik}^{b}\rho\sigma_{i}\sigma_{j}+\alpha_{jk}^{b}\sigma_{j}^{2}\right]\sum_{l=1}^{K}b_{il}^{*}\alpha_{jl}^{b}\right) = 0,\ j\neq i,\ \forall\, k$$

Reordering the above expression yields

$$\underbrace{\left[1-\sum_{l=1}^{K}\left(1-b_{il}^{*}\right)\alpha_{i}^{b}\right]}_{\text{position in asset } i} = \frac{\left(\sum_{l=1}^{2}\alpha_{lk}^{b}\mu_{l}\right)-\theta_{k}}{\gamma\sigma_{i}^{2}\left[\alpha_{ik}^{b}+\rho\frac{\sigma_{j}}{\sigma_{i}}\alpha_{jk}^{b}\right]}-\left(\frac{\sigma_{j}}{\sigma_{i}}\right)^{2}\left[\frac{\alpha_{ik}^{b}\rho\frac{\sigma_{i}}{\sigma_{j}}+\alpha_{jk}^{b}}{\alpha_{ik}^{b}+\rho\frac{\sigma_{j}}{\sigma_{i}}\alpha_{jk}^{b}}\right]\underbrace{\sum_{l=1}^{K}b_{il}^{*}\alpha_{jl}^{b}}_{\text{position in asset } j},\ j\neq i,\ \forall\, k \tag{20}$$

which relates the fraction of asset i to the fraction of asset j that *investors* i hold after trading. Let ϕ and η be two non-negative constants.The system of equations (20) has a solution only if the following $2\times k$ equalities are satisfied

$$\begin{aligned}\left(\sum_{l=1}^{2}\alpha_{lk}^{b}\mu_{l}\right)-\theta_{k} &= \phi\gamma\sigma_{i}^{2}\left[\alpha_{ik}^{b}+\rho\frac{\sigma_{j}}{\sigma_{i}}\alpha_{jk}^{b}\right],\ \forall\, k\\ \alpha_{ik}^{b}\rho\frac{\sigma_{i}}{\sigma_{j}}+\alpha_{jk}^{b} &= \eta\left(\alpha_{ik}^{b}+\rho\frac{\sigma_{j}}{\sigma_{i}}\alpha_{jk}^{b}\right),\ \forall\, k\end{aligned}$$

Define $\zeta=\left(\frac{\sigma_i}{\sigma_j}\right)^2\left(\frac{\eta-\rho\frac{\sigma_j}{\sigma_i}}{1-\eta\rho\frac{\sigma_i}{\sigma_j}}\right)$ and $\lambda=\frac{1}{\mu_i+\zeta\mu_j-\phi\gamma\sigma_i^2\left(1+\rho\zeta\frac{\sigma_j}{\sigma_i}\right)}$. If the above equalities are satisfied then

$$\alpha_{jk}^{b}=\zeta\alpha_{ik}^{b}\quad\text{and}\quad\alpha_{ik}^{b}=\lambda\theta_{k} \tag{21}$$

As a consequence, investors' first order conditions can be solved only if intermediaries issue baskets that are equivalent in their spanning role, as the ratio $\frac{\alpha_{1k}^b}{\alpha_{2k}^b}$ is constant across k. If θ_k varies across intermediaries, the number of shares of assets 1 and 2 varies across baskets.

Intermediary k's problem at equilibrium

Let $b_{\text{int},k}$ denote the fraction of basket k that intermediary k holds after trading. The profits of intermediary k are given by

$$\widetilde{\pi}_{k} = \beta_{k}\left(\sum_{l=1}^{2}\alpha_{lk}^{b}\widetilde{x}_{l}\right)b_{\text{int},k}+\theta_{k}\sum_{l=1}^{2}b_{lk}, \tag{22}$$

where $\beta_k\geq 0$ parameterizes intermediary k's equity stake in basket k, and θ_k denotes the intermediation fee investors pay when trading basket k. In equilibrium all markets clear, and thus, $b_{\text{int},k}^{*}=1-\sum_{l=1}^{2}b_{lk}^{*}$ and the composition of basket k maximizes

$$\mathbb{E}\left[\widetilde{\pi}_{k}^{*}\right] = \beta_{k}\left(\sum_{l=1}^{2}\alpha_{lk}^{b}\mu_{l}\right)\left(1-\sum_{l=1}^{2}b_{lk}^{*}\right)+\theta_{k}\sum_{l=1}^{2}b_{lk}^{*} \tag{23}$$

Provided that $\mathbb{E}\left[\widetilde{\pi}_{k}^{*}\right]\geq 0$, the composition of basket k is described by (21) in equilibrium.